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MEASURED VARIANCES OF $Z_{\rm DR}$, ϕ_{HV}, AND ρ_{HV} FROM SIMULTANEOUS H AND V TRANSMISSIONS

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$Z_{\rm DR}$, ϕ_{dp}, and ρ_{HV} need to be determined at zero lag, namely as if the H and V polarizations were transmitted simultaneously. Instead of actually using simultaneous transmissions, however, the quantities have traditionally been measured using alternate H and V polarizations. The alternating pulse measurements are substantially affected by interpulse Doppler effects that increase both the complexity and uncertainty of the measurements. Because the depolarization effects tend to be small, it is important that the measurement uncertainty be minimized.

Dual-polarization measurements are readily obtained from simultaneous H and V transmissions (Scott, 1999; Brunkow et al., 2000; Scott et al., 2001; paper 5B.2 of this conference). Indeed, this constitutes an optimal way of making the measurements. The dual-polarization quantities are determined from simultaneous returns in the two polarizations and are therefore not contaminated by Doppler effects. The uncertainties of ϕ_{dp} and ρ_{HV} are limited by the fundamental variances of phase and correlation measurements, and the quantities can be determined to within a given uncertainty in less time by not having to average out the Doppler effects.

In this paper we compare observed and theoretical variances of dual-polarization measurements from simultaneous H and V transmissions. The observations were obtained using the New Mexico Tech 3-cm dual-polarization radar, modified to receive circularly polarized transmissions simultaneously in H and V channels (Scott, 1999). The coherent channels of the radar receiver used constant-phase amplitude limiters to achieve the required dynamic range; the consequent loss of the amplitude information caused the observations to have the fundamental variances as their lower bound.

Figures 1 and 2 show sample results. The data were obtained with the antenna pointed in a fixed direction through a localized convective storm in which 45 dBZ reflectivity extended nearly to 10 km altitude (Scott et al., 2001). Figure 1 shows observations at low elevation angle ($0.2°$) through rain and hail falling out of the storm. Figure 2 is from ice-form precipitation (graupel and hail) at about 5 km altitude, just above $0°$C level. The left column in each figure shows the average value of Z_H, $Z_{\rm DR}$, ϕ_{HV}, and ρ_{HV} versus range, with the solid line indicating the range interval of interest. The right column shows the rms fluctuations about the mean in successive 1-μs range gates through the storm. The fluctuations are those of 64-pulse (32 ms) averaged data that comprised the recorded data, and are plotted as a function of the average value of the correlation coefficient $\rho = \rho_{HV}$.

From Bendat and Piersol (1986), the standard deviations of ϕ_{HV} and ρ^2_{HV} are (their equations 9.52 and 9.81)

$$\sigma_\phi \quad = \quad \frac{1}{\sqrt{N}}\,\frac{\sqrt{1-\rho^2}}{\sqrt{2}\rho} \qquad (1)$$

and

$$\sigma_{\rho^2} \quad = \quad \frac{1}{\sqrt{N}}\,\sqrt{2}\rho(1-\rho^2)\;, \qquad (2)$$

where N is the number of independent samples being averaged. The results are from linear signal theory and therefore assume that both the amplitude and phase information is used in the measurements. Kostinski (1994) derived approximate upper bounds for σ_ϕ when ϕ is determined only from the phase and not the amplitude of the signal. The simpler of Kostinski's two results was

$$\sigma_\phi \quad \simeq \quad \frac{1}{\sqrt{N}}\left[\pi - 2\sin^{-1}(\rho)\right]\;, \qquad (3)$$

and was shown to be a good approximation of the more exact (but computationally more complex) result for the upper bound.

For the amplitude-limited data of this study, the rms fluctuations of ϕ_{HV} should lie between lower and upper bounds given by (1) and (3). Two sets of curves describe these bounds on the σ_ϕ graphs. The pair of solid curves show the bounds assuming all 64 samples of the data records were independent ($N = 64$). The dashed curves are the same except the effective number of independent samples in the 64-pulse records was arbitrarily assumed to be 24. The latter set of curves bracket the data points through the rain region (Figure 1), indicating that the measurements were not fully independent from one sample to the next. This is consistent with the short interpulse time interval (0.5 ms) and the fact that the measurements were obtained at horizontal incidence, which minimized Doppler spectral width and increased the time to independence. Somewhat smaller rms fluctuations were observed at the higher elevation angle of Figure 2 ($14.5°$), where the Doppler spectrum width would be increased due to fall speed variation with particle size.

The fluctuations of the other quantities are compared only with their theoretical lower bounds, again assuming $N = 64$ (solid line) and $N = 24$ (dashed line). From the study by Schultz and Kostinski (1997),

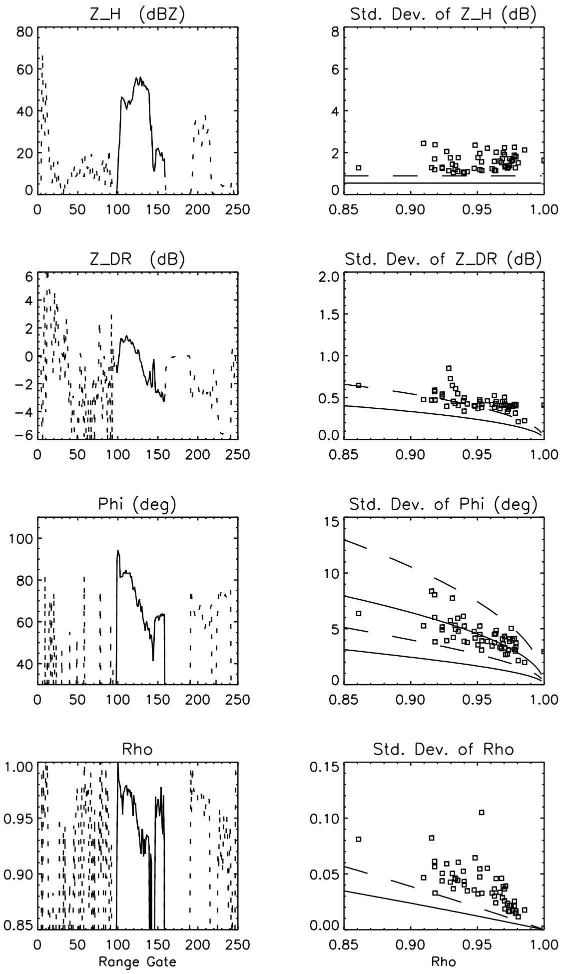

Figure 1. RMS fluctuations of the different radar measurables through the rain region of a storm. The radar antenna was pointed in a fixed direction at $0.2°$ elevation angle for 55 sets of 64-pulse averaged data.

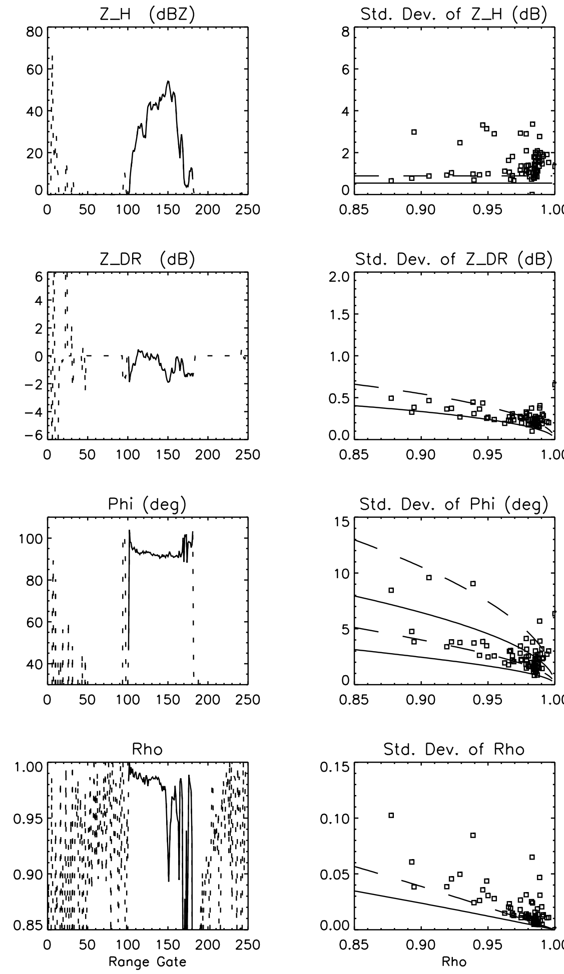

Figure 2. Same as Figure 1, except through ice-form precipitation at about 6 km altitude ($14.5°$ elevation) in the core of the storm. The fluctuations were determined from 23 sets of 64-pulse averaged data.

the standard deviation for logarithmic $Z_{\rm DR}$ measurements is

$$\sigma_{Z_{\rm DR}}({\rm dB}) \quad = \quad \frac{\sqrt{2}}{\sqrt{N}}\,\frac{10}{\ln 10}\,\sqrt{1-\rho^2}\;, \qquad (4)$$

while the fluctuations for ρ_{HV} are

$$\sigma_\rho \quad = \quad \frac{1}{\sqrt{N}}\,(1-\rho^2)\;. \qquad (5)$$

The latter is $\sqrt{2}$ greater than the standard deviation of ρ values derived from measurements of ρ^2, namely $\sigma_{\sqrt{\rho^2}} = \sigma_\rho/\sqrt{2}$. For reflectivity measurements, it is readily shown that

$$\sigma_Z({\rm dB}) \quad = \quad \frac{1}{\sqrt{N}}\,\frac{10}{\ln 10}\;. \qquad (6)$$

This expression is for linearly averaged reflectivity values and will somewhat underestimate the standard deviation of logarithmically detected values.

For each of the measured quantities, the observed fluctuations have the theoretical values as their lower bound. The theoretical variances depend only on the signal statistics and (except for Z_H) are a function of the correlation magnitude ρ between the two signals, increasing as ρ decreases below unity. Non-unity values of ρ are caused by an unpolarized component in the backscattered return, due either to the presence of randomly oriented or shaped particles (or to receiver noise). The presence of an unpolarized component substantially increases the theoretical variances. For $\rho_{HV} = 0.95$, and assuming 64 independent samples, the theoretical rms fluctuations are $1.7°$ for ϕ_{HV}, 0.24 dB for $Z_{\rm DR}$, and 0.01 for ρ_{HV}. For $\rho_{HV} = 0.90$, the fluctuations increase to $2.5°$, 0.34 dB, and 0.024,respectively.

The observed fluctuations reflect the above dependence on ρ but are generally larger than the theoretical values. For the coherently determined quantity ϕ_{HV}, the values appear to be within the upper bound for amplitude-limited measurements assuming the samples were not all independent. These factors also may account for the increased fluctuations in ρ_{HV}. The observed fluctuations in ρ_{HV} are comparable to those reported by Illingworth and Caylor (1991), obtained from a similar number of measurement samples.

$Z_{\rm DR}$ and Z_H were incoherently measured quantities and not affected by the amplitude limiting. The fact that their fluctuations were often larger than predicted implies that the effective number of independent samples was less than estimated number, and/or that other sources of error were present. More detailed measurements would be required to determine the cause of the fluctuations.

Assuming other sources of error to be small, the results illustrate two important points: First, the uncertainties in the coherent quantities ϕ_{HV} and ρ_{HV} should be substantially reduced by determining them in a fully linear manner that utilizes both the amplitude and phase of the data. Such measurements are currently possible using digital IF techniques. Second, the fluctuations would also be reduced through the use of prewhitening techniques discussed by Schultz and Kostinski (1997), which have the effect of making each sample independent. It is important that the fluctuations be minimized for using ϕ_{dp} to estimate rainfall rates, and for quantitative applications of ρ_{HV}.

In closing, we note that the observations were made at 3 cm wavelength. This does not affect the variance analyses but has a significant effect on the variation of $Z_{\rm DR}$ and ϕ_{HV} with range. As can be seen in Figure 1 and as is discussed by Scott et al. (2001), $Z_{\rm DR}$ profiles through rain at this wavelength are substantially biased by differential propagation attenuation. The differential phase shift in passing through the rain was relatively large ($\simeq -45°$) but was also affected by differential phase upon backscatter (evidenced for example in Figure 1 by the sudden increase in ϕ_{HV} at range gate 140). Although obtained at essentially horizontal incidence, the data of Figure 1 were not contaminated by ground clutter due to blockage effects of intervening terrain.

REFERENCES

Bendat, J.S., and A.G. Piersol, 1986: Random Data: Analysis and Measurement Procedures. John Wiley and Sons, Inc., 566 pp.

Brunkow, D., V.N. Bringi, P.C. Kennedy, S.A. Rutledge, V. Chandrasekar, E.A. Mueller, and R.K. Bowie, 2000: A description of the CSU-CHILL National Radar Facility, J. Atmos. Oceanic Tech., 17, 1596-1608.

Illingworth, A.J., and I.J. Caylor, 1991: Co-polar correlation measurements of precipitation, Paper 21.1, Proc. 25th Conf. Radar Meteorol., Amer. Meteorol. Soc., 650-653.

Kostinski, A., 1994: Fluctuations of differential phase and radar measurements of precipitation. J. Appl. Meteor., 33, 1176-1181.

Scott, R., P. Krehbiel, and W. Rison, 2001: The use of simultaneous horizontal and vertical transmissions for dual-polarization radar meteorological observations, J. Atmos. and Oceanic Tech., 18, 629-648.

Scott, R., 1999: Dual-Polarization Radar Meteorology: A Geometrical Approach. Ph.D. Thesis, New Mexico Institute of Mining and Technology, 129 pp.

Schultz, J., and A. Kostinski, 1997: Variance Bounds on the Estimation of Reflectivity and Polarization Parameters in Radar Meteorology, Geosci. and Remote Sensing, 35, 248-255.